

Learning Activities in Mathematics Education: Application of Problem-Based Learning in Power Dominating Set for Electricity Network Optimization Problems



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ABSTRACT: Learning activities in mathematics education play an important role in developing students' analytical and problem-solving skills, especially in applying mathematical concepts to real-world problems. This research aims to apply the Problem-Based Learning (PBL) method to the topic of Power Dominating Set to optimize the electricity network. Power Dominating Set is a concept in graph theory that can be used to minimize the number of control points in the electricity network, thus optimizing the use of resources and increasing the efficiency of the network. By using PBL, students are expected to understand the concept of Power Dominating Set both abstractly and in a practical context relevant to the real world. This research was conducted using an experimental method with two groups of students, namely the experimental group using the PBL method and the control group using the conventional learning method. The results showed that the group that learned with PBL had a deeper understanding and better analytical skills in applying Power Dominating Set to electricity network optimization problems. Discussion of the results showed that the PBL approach is not only effective in improving the understanding of abstract mathematical concepts, but also capable of enhancing students' skills in identifying and solving real-world electricity network problems. Thus, the application of PBL in mathematics education can be an effective alternative to connect abstract mathematical concepts with practical applications, especially in electricity network optimization.

KEYWORDS: Learning activities, Problem Based-Learning, Power Dominating Set, Electricity Network Optimization.

I. INTRODUCTION

Mathematics education has a strategic role in developing critical, logical, and creative thinking skills needed to solve complex problems in various fields of life. One approach that is increasingly popular in mathematics education is problem-based learning (PBL). This approach prioritizes student involvement in the learning process through solving real problems that are relevant to everyday life. In the context of mathematics education, PBL not only improves students' conceptual understanding but also provides opportunities to apply mathematical concepts in practical situations.

One of the interesting topics in mathematics education with direct relevance to real-world applications is Power Dominating Set (PDS). PDS is a subject in graph theory that is used to model and optimize electricity networks. In the design and management of electricity networks, the existence of PDS helps minimize the number of measurements and monitoring devices required to ensure the grid remains efficient and stable. Thus, integrating the PDS concept into problem-based learning has the potential to make a real contribution to the mastery of mathematical concepts while bringing students closer to a practical, solution-oriented understanding. However, in practice, teaching mathematical concepts such as PDS is often still oriented towards traditional approaches. This causes students to tend to understand math procedurally without connecting it to real applications. A problem-based learning approach can be an effective alternative to address this challenge. Therefore, it is important to explore how PBL can be applied in PDS learning to optimize the potential of mathematics education in developing students' analytical and problem-solving skills.

Previous research shows that PBL can significantly improve students' critical thinking skills in various subjects, including mathematics. This research reflects the relevance and effectiveness of Problem-Based Learning (PBL) in various learning contexts to improve students' critical, creative, and metacognitive thinking skills. Syamsudin (2020), Sumadun and Dwikoranto (2022) showed that the application of PBL in physics learning significantly improved students' critical thinking skills by providing real-world challenges that integrate theoretical concepts. Similar research by Shamdas et al. (2024) confirmed that PBL not only has an impact on improving science learning outcomes, but also strengthens students' metacognitive knowledge through a structured and

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interactive learning approach. Pangaribuan (2022) extended the application of PBL by showing its success in helping students identify the structure and linguistic elements of explanatory texts, confirming the flexibility of this model for different subjects.

In addition, Zulkarnaen et al. (2022) found that PBL promotes the development of students' creative thinking skills through a collaborative process of exploration and problem solving. Islamiati et al. (2024) further emphasized the interaction between the PBL model and individual learning styles, noting that this approach can be adapted to meet the thinking needs of diverse students. Erawanto (2016) supported these findings by highlighting the development of PBL-based learning modules that effectively enhance students' creativity. Recent research by Marsidi et al. (2023) combined the PBL framework with the Resolving Dominating Set (RDS) technique in STEM learning and showed that this combination not only improved students' creative thinking skills, but also strengthened their ability to solve complex problems, such as disaster prediction systems. Purwanto et al. (2023) support project-based approaches and PBL as relevant strategies for technical subjects such as basic electricity and electronics in vocational schools. Meanwhile, Syamsudin (2020) emphasizes the importance of inquiry-based learning tools as a complement to the PBL model and provides insight into the use of virtual laboratories to enrich the learning experience.

Overall, this research demonstrates that PBL is a versatile and effective approach for enhancing a wide range of cognitive and learning skills, with applications in mathematical, science, linguistic, and engineering learning contexts. This provides a solid foundation for exploring the application of PBL to mathematical concepts such as Power Dominating Set in optimizing electricity networks. The concept of power domination in graph theory has attracted widespread attention due to its relevance in real-world applications such as electricity networks optimization. The initial study by Zhao et al. (2006) laid the foundation for research by defining the basic framework of power domination in graphs, discussing propagation rules, and investigating various fundamental properties of these sets. This research paved the way for further exploration of the topic. Future research expands the scope of this concept. Yuliana et al. (2019) investigated the value of power domination numbers on graph operations such as corona products and join graphs. This study provides important insights into the influence of graph structure on power domination properties. Zhao et al. (2020, 2021) continued the exploration by studying power domination on generalized Petersen graphs and formulating solutions to certain conjectures, showing significant progress in understanding the complex dynamics of certain graphs. Benson et al. (2018) highlighted the Nordhaus-Gaddum aspect of power domination, which relates parameter values in graphs to their complements, providing new insights into the symmetric relationship between graph structures. Anderson and Kuenzel (2022) focused on cubic graphs and Cartesian products, revealing the relationship of power domination parameters in graphs with certain regular structures, while Lu et al., (2020) investigated claw-free regular graphs, extending the understanding of graphs with additional structural constraints. The study by Chang et al., (2012) introduces the concept of generalized power domination, broadening the scope of the research by adding generalization elements relevant to broader applications. The recent study by Bläsius and Göttlicher (2023) provides an efficient algorithm for finding power dominating sets, demonstrating the focus on computational efficiency in the practical application of this concept.

This research contributes to the development of the field of mathematics education by integrating the concept of Power Dominating Set (PDS) from graph theory into the Problem-Based Learning (PBL) approach, a perspective that has not been widely explored. Studies on PDS so far have focused more on the mathematical aspects and its application in network management, as discussed by Yuliana et al. (2019) on the properties of PDS on graph combinations such as corona product and join graphs, and by Zhao et al. (2020) who highlighted the application of PDS on generalized Petersen graphs. This research goes further by linking problem-based mathematics learning with PDS concepts in the context of power grid optimization, providing a new approach to develop students' ability to link theoretical mathematics with practical applications.

Previous research, such as Zhao et al. (2006) who outlined the theoretical basis of PDS in graphs, and Benson et al. (2018) who investigated the Nordhaus-Gaddum problem for PDS, have provided an important foundation in understanding the complexity and technical applications of this concept. However, a limitation in such research lies in the lack of exploration of how the concept of PDS can be effectively taught in formal educational settings, especially through innovative learning strategies such as PBL. This research fills that gap by offering a new approach in learning design that is not only oriented towards theory, but also towards students' problem-solving skills.

Furthermore, other studies such as Anderson & Kuenzel (2022) who studied PDS in cubic graphs and Cartesian products, and Lu et al. (2020) who explored PDS in claw-free regular graphs, show the potential for wide applications of PDS in graph theory. Similarly, efforts to develop efficient algorithms for PDS, such as those proposed by Bläsius & Göttlicher (2023), demonstrate the relevance of PDS in technology and computing. However, these studies remain in the realm of algorithmic and technical development without regard to the educational dimension.

Therefore, this research makes new contributions in two main ways: first, by exploring the application of PDS concepts in problem-based mathematics learning to support students' conceptual understanding as well as applicative skills; second, by introducing a pedagogical dimension to PDS studies that have previously focused more on theoretical and technical development. Thus, this research bridges the gap between advanced graph theory and its application in education, while offering an innovative learning model to enhance the relevance and effectiveness of mathematics education.

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Overall, these studies show rapid developments in power domination theory, covering both theoretical and algorithmic aspects, while expanding its potential applications in network systems and optimization. These ideas provide a solid foundation for further studies, particularly on the integration of these concepts in mathematics education and simulation-based problem learning. However, research explicitly linking problem-based learning with PDS in mathematics education is still very limited. Most studies still focus on the theoretical aspects of PDS or technical applications in electricity networks engineering, without highlighting how these concepts can be effectively taught at the secondary or tertiary education level. Thus, there is a significant research gap to examine how the application of PBL in PDS learning can improve students' understanding of mathematical concepts as well as their application relevance. Based on the above background, this research is focused on the following main questions:

1. What is the problem-based learning research framework for the Power Dominating Set concept for electricity networks optimization?
2. How are the learning activities in problem-based learning on the concept of Power Dominating Set for electricity networks optimization?

This study aims to answer these questions in order to provide theoretical and practical contributions to the development of relevant and applicable mathematics learning strategies. Thus, the results of this study are expected to be a reference for educators and researchers to integrate the PBL approach in teaching mathematics, especially on topics that have direct application in real life, such as Power Dominating Set.

The syntax of problem-based learning begins with (1) presenting the problem, where students are given a contextual problem and asked to understand the content of the story; (2) discussing the problem, where students are asked to find facts and plan a solution; (3) returning to the group to discuss the results of the work and share with friends; (4) independent learning, where students are asked to guess the answer, test, and conclude the results of the work; (5) presenting the results of the discussion; and (6) reviewing the results of the work (Erawanto, 2016). Indicators of creative thinking skills: fluency, flexibility, originality, and elaboration (Marsidi et al., 2023).

II. METHOD

This study uses a qualitative method that focuses on the development of a problem-based learning framework (Problem-Based Learning / PBL) to teach the concept of Power Dominating Set in the context of electricity networks optimization. The research began with the collection of relevant literature and literature review, including graph theory, Power Dominating Set, the application of PBL in mathematics education, and electricity networks optimization. Based on the results of the review, the learning framework was developed using the 4D model (Define, Design, Develop, and Disseminate).

In the Define stage, researchers identify learning needs and analyze the gap between mathematical theory and its practical implementation. In the Design stage, the problem-based learning module is designed to integrate the Power Dominating Set with the electricity networks optimization problem, including exploration, analysis, and problem-solving activities. Furthermore, the Develop stage is carried out by compiling teaching materials, student worksheets, and PBL implementation guidelines which are then tested in small groups to obtain input. In the final stage, namely Disseminate, the enhanced modules are disseminated through teacher training or educational seminars to test their relevance and effectiveness in various learning contexts. This approach aims to connect abstract mathematical concepts with real-world applications, while improving students' analytical abilities in solving real problems in the field of electricity networks.

III. RESULTS & DISCUSSION

The research results in this article are divided into two subchapters that discuss the problem-based learning (PBL) framework and the integration of the PBL framework with evaluation strategies in the application of the power dominating set concept to electricity networks optimization. This research emphasizes the importance of mathematical modeling and optimization in real-world contexts, particularly in improving the efficiency and reliability of electricity networks systems.

The first subsection discusses how the PBL framework encourages students to be actively involved in identifying and solving mathematical problems related to power dominating sets. Students are guided to integrate their mathematical understanding with the context of real-world problems to develop innovative solutions for electricity networks optimization. Activities include problem analysis, group discussion, and development of mathematical models to solve the problem.

The second section focuses on assessment strategies designed to measure students' creative thinking, problem solving, and adaptability in the context of PBL. The assessment evaluates students' fluency, flexibility, originality, and decomposition of ideas in addressing challenges related to electricity networks optimization. The assessment process is designed to capture both the process and the outcomes of learning, so that students not only generate solutions, but are also able to explain and refine their approach based on the feedback they receive.

This research highlights the practical relevance of integrating mathematical concepts such as power dominating sets with a problem-based learning approach. By simulating real-world challenges in electricity networks optimization, this research equips

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students with the analytical and creative skills needed to deal with complex problems. In addition, this research provides a foundation for the development of innovative teaching practices that link abstract mathematical theories with real-world applications, thereby increasing student engagement and deep understanding.

3.1 PBL Learning Framework

This research aims to develop a problem-based learning (PBL) model that can be applied in the context of mathematics education, specifically in teaching the concept of power dominating sets. This concept is relevant to solving electricity networks optimization problems, such as determining the minimum number of sensors needed to efficiently monitor the network. This complex electricity networks optimization problem is an opportunity to connect abstract theories in mathematics with practical applications in the real world.

Problem-based learning was chosen because this approach focuses on developing students' critical, creative, and collaborative thinking skills. In this model, students are faced with real problems that do not have a single solution, so they must analyze, design strategies, and implement solutions independently or in groups. This approach provides students with a deep learning experience, where they not only learn mathematical theories but also understand how these theories can be applied in real situations.

The power dominating set concept used in this research is part of graph theory, where vertices in a graph are chosen to dominate all other vertices through certain graph relationships. In the context of electricity networks, this concept can be applied to determine the optimal position of sensors or other control devices to ensure system efficiency and reliability. The implementation of this concept requires a deep understanding of graph theory as well as the ability to relate the concept to the technical needs in electricity networks optimization.

In addition to its relevance to the real world, problem-based learning is also in line with modern educational goals that emphasize the development of 21st century capabilities, such as problem solving, communication, collaboration, and technological literacy. By integrating the concept of power dominating sets in learning, students are invited to understand the role of mathematics as a tool to solve real challenges. This approach also provides space for students to explore their creativity in developing innovative solutions.

This research was designed to address the need for more meaningful and applicable learning. In the context of mathematics education, students often find it difficult to understand abstract concepts due to the lack of connection with practical applications. The PBL model applied in this study aims to bridge the gap by bringing real problems into the classroom. In this way, students not only understand mathematical concepts such as power dominating sets, but also gain experience in implementing these concepts to solve relevant problems.

This problem-based learning framework is designed to actively engage students through a gradual learning process, from understanding the problem, analyzing facts, to designing and presenting solutions. In each stage of learning, students are encouraged to think critically, work together with groups, and take the initiative in self-learning. This process is expected to not only improve concept understanding, but also build the skills needed to face real-world challenges.

The application of this learning model also aims to increase student involvement in learning. By being exposed to contextually relevant problems, students will be more motivated to learn because they can see how mathematics is used to solve the problems they face. This motivation, in turn, will encourage students to actively participate in every stage of learning, thus providing a meaningful learning experience.

This research is expected to be a reference for the development of other learning methods in mathematics education, especially to connect theory with practical applications. By presenting real problems such as electricity networks optimization, this model not only helps students understand mathematical concepts in depth but also equips them with relevant skills to face the professional world.

Next, we will explain in more detail the syntax of problem-based learning used in this study, starting from the problem presentation stage to the evaluation or review of work results stage. The problem-based learning syntax used in this study consists of six stages, namely problem presentation, problem discussion, group work, self-study, presentation of results, and reviewing work results (See **Figure 1**).

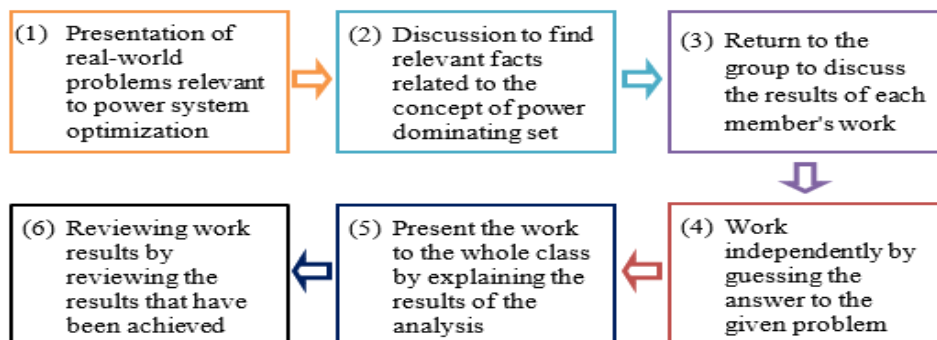


Figure 1. Syntax Framework of Problem Based Learning Model.

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Stage 1: Problem Presentation

The first stage begins with the presentation of a contextual problem derived from a real-world situation. Students are given a scenario related to the electricity networks, such as how to determine the minimum number of sensors needed to monitor the electricity networks without losing control of the entire system. This problem is presented through stories, illustrations, or case simulations to help students understand the context of the problem. The presentation of the problem is designed to capture students' attention and provide a challenge that stimulates their desire to explore solutions.

At this stage, students are asked to read, understand, and identify key elements of the given problem. For example, students may be instructed to observe the complex structure of the electricity networks and relate it to the concept of graph theory in mathematics in **Figure 2**. This sets the stage for the next step, where students begin to plan problem-solving strategies.

Stage 2: Problem Discussion

After understanding the problem, students enter the group discussion phase. In this stage, they work together to identify relevant facts, such as the basic concept of the power dominating set and how it can be applied to minimize the number of sensors in the electricity networks. Students are also asked to formulate initial hypotheses and develop problem-solving strategies.

This discussion process involves an exchange of ideas among group members. Students are encouraged to use critical and creative thinking skills as they explore different possible solutions. In addition, this discussion helps them strengthen their understanding of relevant concepts through peer learning. At this stage, students can graphically represent the housing network as shown in **Figure 3**. This discussion becomes an important foundation in the process of developing group solutions.

Stage 3: Back to Group

After the initial discussion, students return to their groups to discuss their individual work and share the information they have gathered. The purpose of this stage is to bring together the different perspectives of each group member. In this process, students evaluate the strategies they developed earlier and adjust them based on input from their peers.

At this stage, students begin to formulate a more mature interim solution. They also effectively distribute tasks among group members to optimize the problem-solving process. This stage is one of the most important elements in developing students' collaborative skills.

Stage 4: Self-study

The next stage is independent study. In this stage, students are asked to work individually to explore and refine their solutions. They speculate about the answers to the problem, perform mathematical calculations or simulations, and test the solutions they design. In the context of power dominating sets, students model the electricity networks using a graph and determine the nodes that serve as optimal controllers.

Independent study allows students to hone their analytical and problem-solving skills. In addition, this stage gives students the opportunity to explore solutions in more depth and ensure that the results of their analysis are mathematically valid. This process also helps students develop lifelong learning skills, which is one of the main goals of problem-based education.

Stage 5: Presentation of Results

After completing the individual and group tasks, students present their work to the whole class. In this presentation, students are asked to explain the solution they developed, including the strategies used, the results of the analysis, and the mathematical justification for the solution. The presentation also includes the presentation of logical arguments to support their choice of solution.

This presentation process trains students to organize and communicate ideas in a systematic and professional manner. In addition, they receive feedback from instructors and peers that can help them refine their solutions. This stage not only develops students' communication skills, but also helps them learn from the perspectives and experiences of other groups.

Stage 6: Reviewing the Work

The last stage is reviewing the results of the work. In this stage, students and lecturers review the results that have been achieved, both in terms of the process and the solutions produced. This review process aims to evaluate the effectiveness of the strategies used, the strengths and weaknesses of the proposed solutions, and the lessons learned for future improvements.

The review gives students the opportunity to reflect on their learning experience and improve their understanding of relevant concepts. In addition, it also helps students to identify areas that require improvement, both in terms of technical skills and collaboration skills.



Figure 2. Housing Electrical Networks.

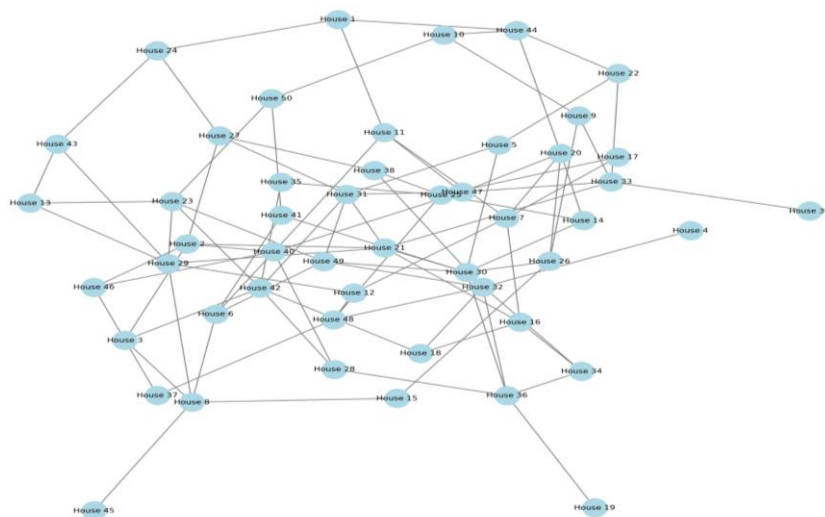


Figure 3. Graph Representation of Housing Network with 50 Vertices.

3.2 Learning Outcomes and Objectives

The learning outcomes achieved in this research show that students are able to use the concept of power dominating set effectively to solve electricity networks optimization problems. Students not only understand the basic theory of power dominating set in the context of graph theory, but are also able to apply the concept in real problems, such as determining the minimum number of sensors needed to monitor the electricity network efficiently without reducing system reliability. This ability shows an increase in students' understanding of mathematical concepts and analytical skills.

In addition to technical mastery, this research also succeeded in improving students' creative thinking skills. In the problem-based learning process, students are encouraged to explore various alternative solutions, design innovative strategies, and develop new ideas relevant to the electricity networks optimization problem. This process helps students to not only focus on one solution, but also consider various approaches that allow them to find more effective and efficient solutions.

This increase in creative thinking skills was seen in various stages of learning, especially in the group discussion and self-study stages. Students showed the ability to integrate mathematical concepts with applicative approaches, conduct simulations, and validate the solutions they designed. Active involvement in every stage of learning also helps students to be more confident in communicating their ideas, both in writing through reports and orally through presentations.

With this achievement, problem-based learning not only provides students with a strong conceptual understanding of the power dominating set, but also equips them with critical, collaborative and creative thinking skills that are essential to face challenges in the real world. These results suggest that this learning model is effective in creating meaningful and relevant learning experiences, while preparing students to apply mathematics in practical contexts in the future. Objectives of this research are as follows.

- a. Integrates theory and application: Helps students understand the relationship between mathematical theory, especially power dominating sets, and real applications in electricity networks optimization.

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- b. Improve analytical skills: Trains students to identify important elements in the electricity networks, model the problem in graph form, and determine the optimal solution.
- c. Hone creative thinking skills: Encourage students to explore various innovative approaches in solving electricity networks optimization problems.
- d. Building collaborative skills: Engage students in group discussions to design and evaluate problem-solving strategies together.
- e. Train communication skills: Provide students with the experience to organize and present their solutions in a logical and convincing manner, both orally and in writing.
- f. Motivates contextualized learning: Connects math learning to real-world problems, thus increasing motivation and relevance of learning for students.

3.3 PBL Framework and Assessment in Problem Solving Disaster Monitoring and Prediction System

The Problem-Based Learning (PBL) framework used in this study is designed to enhance students' creativity in understanding and applying the concept of power dominating set for electricity networks optimization. In this framework, the indicators of student creativity are divided into four main categories, namely fluency, flexibility, originality, and elaboration. Each indicator has sub-indicators designed to assess different aspects of students' ability to solve problems relevant to the real world.

Table 1 describes these indicators and their sub-indicators, as well as the test materials or assessment instruments used to measure students' abilities in the problem-based learning process in the context of electricity networks optimization.

Table 1. Student Creativity Indicators in the Context of Power Dominating Set.

Indicator	Sub-Indicator	Test Materials/Assessment Instrument
Fluency	<ul style="list-style-type: none"> - Number of Ideas: Evaluate the number of ideas or solutions generated by students in solving electricity networks optimization problems using power dominating sets. - Diversity of Concepts: Assess the ability of students to generate a variety of ideas or concepts related to the application of power dominating sets in electricity networks. 	<ul style="list-style-type: none"> - Give a problem about power dominating set based electricity networks optimization. Students record ideas/solutions within a certain time limit. - Give a scenario related to the importance of electricity network optimization for energy efficiency. Students explore ideas in group discussions.
Flexibility	<ul style="list-style-type: none"> - Approach Selection: Assess students' ability to switch between approaches or strategies in solving electricity networks optimization problems. - Idea Adaptation: Evaluate the extent to which students are able to adapt their ideas/solutions to changes in the electricity networks context. 	<ul style="list-style-type: none"> - Present an electricity network problem that requires adaptation of the solution. Observe the approach students choose. - Introduce new challenges or sudden changes as students work on the project. Observe how they adapt ideas or solutions.
Originality	<ul style="list-style-type: none"> - Idea Creativity: Assess the level of novelty and creativity of ideas/solutions generated by students in the context of electricity networks optimization using the power dominating set technique. - Technological Innovation: Evaluate the extent to which students can propose innovative technology-based ideas to optimize the electricity networks. 	<ul style="list-style-type: none"> - Provide a concrete problem that requires an innovative solution based on the power dominating set. Assess the novelty of students' ideas. - Propose a project with high technology requirements. Ask students to propose technology-based ideas to optimize the electricity networks.
Elaboration	<ul style="list-style-type: none"> - Detailed Explanation: Assess the student's ability to provide a detailed and elaborate explanation of the idea/solution. - Concept Development: Evaluate the extent to which students can develop and expand ideas/concepts in depth, especially in the implementation of power dominating sets in electricity networks. 	<ul style="list-style-type: none"> - Students present solutions or ideas related to electricity networks optimization. Assess their presentations or written reports. - Assign a project in the context of power dominating set-based electricity networks. Observe how students develop and extend ideas in the project report.

Based on **Table 1**, it can be seen that each indicator not only evaluates the final result but also the process that students go through in developing solutions to electricity network optimization problems. The fluency indicator focuses on students' ability to generate various ideas and solutions, while flexibility assesses the adaptability and exploration of the strategies used. Originality

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measures students' level of creativity and innovation in designing solutions, and elaboration evaluates their ability to provide detailed explanations and develop concepts in depth.

This assessment aims to ensure that students not only understand basic mathematical concepts but are also able to relate them to practical applications, as well as develop solutions that are relevant to the needs of electricity networks optimization. This framework helps in creating meaningful learning by integrating mathematical theories into complex real-world situations.

IV. CONCLUSIONS

This study aims to examine the application of Problem-Based Learning (PBL) in teaching the concept of Power Dominating Set (PDS) as part of mathematics education that is oriented towards solving real problems, especially for electricity networks optimization. Based on the results of the research and analysis, it can be concluded that the PBL approach provides a number of significant benefits in the learning process, including:

1. Improved Conceptual Understanding

PBL effectively encourages students to understand PDS concepts not only theoretically, but also their application in real life, especially in the context of electricity networks optimization. With a problem-based approach, students are more actively involved in learning and have a deep understanding of how mathematical concepts are applied to solve real problems.

2. Development of Problem Solving Skills

Through scenarios and case studies presented in the PBL learning model, students are able to develop critical, analytical and creative thinking skills in solving problems. This shows that PBL has the potential to be an effective learning strategy to train essential 21st century skills.

3. Relevance to Real Applications

The integration of the PDS concept in mathematics learning provides a learning experience that is relevant to the real world, so that students can see a direct relationship between the material learned and the challenges of the industrial and technological world, such as the design and management of electricity networks.

4. Student Motivation and Engagement

The PBL approach increases students' learning motivation because they feel directly involved in a meaningful learning process. The contextualized learning situation also helps students understand the importance of learning math in solving real problems.

Overall, the application of PBL in mathematics learning, especially on the concept of PDS, shows great potential in improving the quality of learning by emphasizing relevance, active participation, and practical application. This research contributes both theoretically and practically, by broadening the horizons of innovative learning strategies and providing implementation guidelines that can be applied at various levels of education.

Open Problem:

As a follow-up step, similar research can be developed by expanding the scale of participants or covering other topics in mathematics that have real-world applications. In addition, a long-term evaluation of the impact of PBL in mathematics teaching also needs to be conducted to measure its effectiveness on an ongoing basis. Thus, it is hoped that this approach can be more widely adopted in the education system.

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REFERENCES

- 1) Anderson, S. E., & Kuenzel, K. (2022). Power domination in cubic graphs and Cartesian products. *Discrete Mathematics*, 345(11), 113113. DOI: <https://doi.org/10.1016/j.disc.2022.113113>
- 2) Benson, K. F., Ferrero, D., Flagg, M., Furst, V., Hogben, L., & Vasilevska, V. (2018). Nordhaus–Gaddum problems for power domination. *Discrete Applied Mathematics*, 251, 103-113. DOI: <https://doi.org/10.1016/j.dam.2018.06.004>
- 3) Bläsius, T., & Göttlicher, M. (2023). An efficient algorithm for power dominating set. *arXiv preprint arXiv:2306.09870*. DOI: <https://doi.org/10.48550/arXiv.2306.09870>
- 4) Chang, G. J., Dorbec, P., Montassier, M., & Raspaud, A. (2012). Generalized power domination of graphs. *Discrete applied mathematics*, 160(12), 1691-1698. DOI: <https://doi.org/10.1016/j.dam.2012.03.007>
- 5) Erawanto, U. (2016). Pengembangan modul pembelajaran berbasis masalah untuk membantu meningkatkan berfikir kreatif mahasiswa. *JINoP (Jurnal Inovasi Pembelajaran)*, 2(2), 427-436. DOI: <https://doi.org/10.22219/jinop.v2i2.2629>
- 6) Islamiati, A., Fitria, Y., Sukma, E., Fitria, E., & Oktari, S. T. (2024). The Influence of The Problem Based Learning (PBL) Model and Learning Style on the Thinking Abilities. *Jurnal Penelitian Pendidikan IPA*, 10(4), 1934-1940. DOI: <https://doi.org/10.29303/jppipa.v10i4.6219>

Learning Activities in Mathematics Education: Application of Problem-Based Learning in Power Dominating Set for Electricity Network Optimization Problems

- 7) Lu, C., Mao, R., & Wang, B. (2020). Power domination in regular claw-free graphs. *Discrete Applied Mathematics*, 284, 401-415. DOI: <https://doi.org/10.1016/j.dam.2020.03.055>
- 8) Marsidi, M., Sunardi, S., Susanto, S., & Suwito, A. (2023). RBL-STEM Learning Framework: Improving Students' Creative Thinking Skills in Solving Natural Disaster Monitoring and Prediction System Problems Using The Resolving Dominating Set Technique. *CGANT JOURNAL OF MATHEMATICS AND APPLICATIONS*, 4(2), 123-140. DOI: <https://doi.org/10.25037/cgantjma.v4i2.106>
- 9) Pangaribuan, F. R. (2022). Efforts to Improve the Ability to Identify the Structure and Linguistic Elements of the Explanation Text using the Problem Based Learning (PBL) Model for VIII Class Students. *Indonesian Journal of Education and Mathematical Science*, 3(1), 29-33. DOI: <https://doi.org/10.30596/ijems.v1i2.5470.s780>
- 10) Purwanto, P., Tamrin, A., & Suharno, S. (2023). Analisis pengembangan sintak model pembelajaran project-based learning pada mata pelajaran dasar listrik dan elektronika di sekolah vokasional. *Jurnal Ilmiah Pendidikan Teknik dan Kejuruan*, 16(1), 1-10. DOI: <https://doi.org/10.20961/jiptek.v16i1.68064>
- 11) Samadun, S., & Dwikoranto, D. (2022). Improvement of student's critical thinking ability sin physics materials through the application of problem-based learning. *IJORER: International Journal of Recent Educational Research*, 3(5), 534-545. DOI: <https://doi.org/10.46245/ijorer.v3i5.247>
- 12) Shamdas, G. B., Laenggeng, A. H., Ashari, A., & Fardha, R. (2024). The Influence of the Problem-Based Learning Model on Metacognitive Knowledge and Science Learning Outcomes. *Jurnal Penelitian Pendidikan IPA*, 10(3), 1383-1395. DOI: <https://doi.org/10.29303/jppipa.v10i3.5444>
- 13) Syamsudin, F. I. (2020). Pengembangan perangkat pembelajaran eksperimen listrik statis berbasis inkuiri terbimbing laboratorium menggunakan phet colorado. *Jurnal Materi dan Pembelajaran Fisika*, 10(2), 68-75. DOI: <https://doi.org/10.20961/jmpf.v10i2.43132>
- 14) Yuliana, I., Agustin, I. H., & Wardani, D. A. R. (2019, April). On the power domination number of corona product and join graphs. In *Journal of Physics: Conference Series* (Vol. 1211, No. 1, p. 012020). IOP Publishing. DOI: <https://doi.org/10.1088/1742-6596/1211/1/012020>
- 15) Zhao, M., Kang, L., & Chang, G. J. (2006). Power domination in graphs. *Discrete mathematics*, 306(15), 1812-1816. DOI: <https://doi.org/10.1016/j.disc.2006.03.037>
- 16) Zhao, M., Shan, E., & Kang, L. (2020). Power domination in the generalized Petersen graphs. *Discussiones Mathematicae Graph Theory*, 40(3), 695-712. DOI: <https://doi.org/10.7151/dmgt.2137>
- 17) Zhao, M., Shan, E., & Kang, L. (2021). On a Conjecture for Power Domination. *Graphs and Combinatorics*, 37(4), 1215-1228. DOI: <https://doi.org/10.1007/s00373-021-02307-8>
- 18) Zulkarnaen, Z., Suhirman, S., Hidayat, S., Prayogi, S., Sarnita, F., Widia, W., & Verawati, N. N. S. P. (2022). The Effect of Problem Based Learning Model on Students' Creative Thinking Ability. *Jurnal Penelitian Pendidikan IPA*, 8(1), 379-382. DOI: <https://doi.org/10.29303/jppipa.v8i1.1307>



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