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Pedagogical Practice- A Study about Designing Mathematics Investigation Tasks for Senior Students in Australia

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ABSTRACT: This paper explored the potential of Realistic Mathematics Education (RME) to support the design of a framework of Investigation- a new curriculum for senior mathematics in Australia. Qualitative and quantitative methods were used in this study. Recognising that RME learning characteristics connect with the components of Investigation, an interview was applied to explore teachers' interpretations of REM potential and transform these into the Investigation model. Pedagogical approaches like the RME, Piaget's constructive theory and Vygotsky's sociocultural theory were embedded in the design of the draft framework. Data about students' confidence in doing the tasks and the challenges of doing the Investigation were collected from a survey after the first Investigation. The survey findings showed that students understand the Investigation's aim, feel confident in doing the task, and work effectively with the Investigation framework with average results of over 80%. Teachers' feedback and evaluation were also collected after the first Investigation via an interview. Then, the second Investigation was designed based on the data from the survey, interview, and the academic results of the first Investigation. Data were analysed and compared, along with the academic results from the second Investigation, combined with students' and teachers' understanding of the Investigation in the Victorian Mathematics Curriculum in Australia, finally, the framework was revised. This study provides a new framework for the new section of the Mathematics Curriculum in Australia, highlighting the significance of the connection between classroom learning and realistic situations.

KEYWORDS: Mathematics Learning Realistic Mathematics Education Investigation Task Model of Investigation Framework of Investigation

I. INTRODUCTION

Mathematics is a compulsory subject taught at all levels of education, from primary school to higher education. Solving mathematical problems and improving mathematical skills improves students' cognitive skills over time. Mathematics has become a basic capital for life, especially when students understand algorithms and problems more deeply, they can decode facts and more easily solve problems. For mathematics learning, students are supposed to master basic abilities, including the ability of students to use the mathematical concepts they have learned to solve practical problems ([1]), the ability to mathematize real situations, and the ability to explain, and reflect on and verify mathematical results in actual situations, which are essential processes of solving mathematical problems ([2]).

In fact, many students feel afraid of learning mathematics and facing difficulties. The research found that some students have mathematic anxiety and are reluctant to do mathematics subjects ([3]), they lack interest in mathematics and gradually fail it. One reason is that learning was oriented toward achieving the curriculum target ([1]), learning activities were mainly designed based on teacher-centred, and students did not have enough opportunities to further explore their efforts. Students felt excluded from the content of their lives. Questions such as "Why are we learning this? Why do we have to know how to do this? When are we ever going to use this outside of class?" might sound familiar to teachers. Students in mathematics classes commonly ask these questions when they are unable to make connections between what they are learning in the classroom and their daily lives ([4]). Another reason is that most students thought mathematics was difficult and boring ([5]). Studying mathematics, covering complex formulas and examples of diverse content, and using problem-solving skills and strategies to solve difficult problems, makes mathematics learning more complicated. Usually, general mathematical class teaching, like teacher-centred, makes it harder for students to develop in mathematics learning if they have fewer opportunities to express their ideas and figure out mathematical concepts based on their understanding.

To improve this situation, students need to have a desire to learn; therefore, the approach to learning should remain eager, motivated, and related. Ease of learning can be experienced if the learning content and context are relevant to students' daily activities. The

Australian Curriculum, Assessment and Reporting Authority ([6]) supported investigative approaches to promote deep understanding and connections between mathematical ideas. Mathematical Investigation, as a new section is added to the Victorian Curriculum and Assessment Authority (VCAA) ([7]) from 2023 to 2027 for Australian senior students who study mathematics as their subject. The Investigation is a task that comprises practical or theoretical contexts or scenarios based on the content students have learned and targets key mathematics knowledge, problem-solving skills, and thinking skills. It provides opportunities for students to develop concepts, learning skills and processes for some realistic problems. As the VCAA does not provide details about how to design the curriculum of Investigation, it is worth exploring setting the framework and designing an effective pedagogical practice in the Investigation. Realistic mathematics education (RME) helps students to visualize and put mathematics into real-life contexts. It encourages discussion and depth of understanding of mathematics knowledge.

II. LITERATURE REVIEW

Realistic Mathematics Education (RME) is a domain-specific instruction theory for mathematics, which is developed in the Netherlands. RME is rooted in Freudenthal's ideas which said that mathematics is a part of human life ([8]), he believes that students should have the opportunity to rediscover mathematics by managing and dealing with real-world situations or mathematical relationships and processes that are meaningful to them ([3]), he emphasised actions and activity of mathematical thinking, rather than formal mathematics. RME builds on students' informal sense-making in response to imaginable and meaningful contexts ([9]). RME helps to fix and improve students' understanding of mathematics concepts ([10]; [11]; [12]), it is one of the approaches that address problems caused by traditional and abstract mathematics learning ([13]). RME is an engaging, problem-solving approach to learning and teaching secondary mathematics. RME curriculum builds deep, long-term mathematical understanding by starting from contexts that students can make sense of ([14]). RME is applied and explored outside the Netherlands, such as in America, China, Africa, Indonesia, and Vietnam.

The key ideas in RME have attracted the attention of researchers, such as Val Hiele's level of learning mathematics, he concluded that the characteristics of RME learning are carried out through three levels ([15]):

(1) a pupil reaches the first level of thinking as soon as he can manipulate the known characteristics of a pattern that is familiar to him/her; (2) as soon as he/she learns to manipulate the interrelatedness of the characteristics he/she will have reached the second level; (3) he/she will reach the third level of thinking when he/she starts manipulating the intrinsic characteristics of relations(p1-4).

Freudenthal's didactical phenomenology states that learning should start from a contextual problem, five basic characteristics of RME ([11]) are summarized as follows:

- *phenomenological exploration or the use of contexts;*
- *the use of models or bridging by vertical instruments;*
- *the use of students' own productions and constructions or students' contributions;*
- *the interactive character of the teaching process or interactivity; and*
- *the intertwining of various learning strands (p174-176).*

In the process of learning, students are provided the opportunities to go through stages referred to in RME as horizontal and then vertical mathematization (see Figure 1). Horizontal mathematization is when students use their informal strategies to describe and solve a real contextual problem, and then to design a mathematical model, vertical mathematization occurs when students' informal strategies lead them to solve the problem using mathematical language or to find a suitable algorithm ([16]).

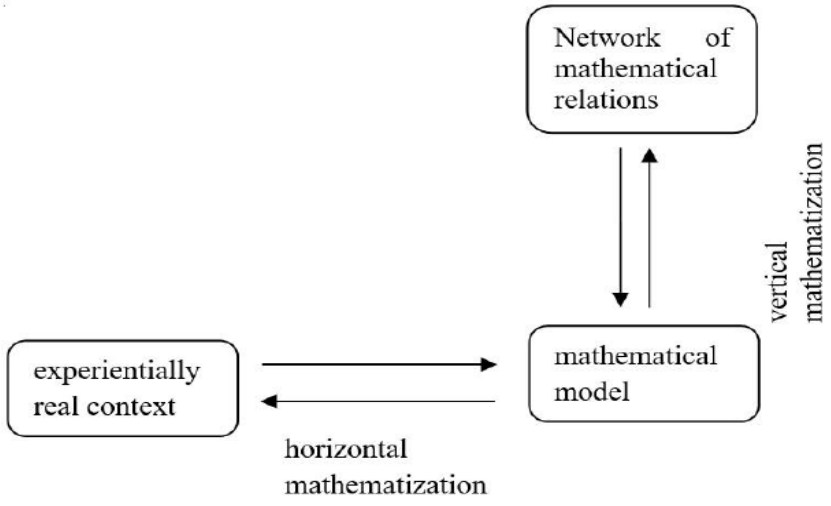

Figure 1. Horizontal and vertical mathematization (based on [17], p. 54)

The aim of RME is to provide an innovative learning approach for teachers and students, which emphasises that mathematics must be associated with real life using real-world context as a starting point of learning ([18]). RME emphasises a 'realistic' situation, working to tackle the complex cohesion between multiple real worlds and mathematical concepts, RME focuses on students' understanding of 'realistic' contexts and appears to be well prepared to tackle similar problems ([14]). So, what is a 'realistic' situation? In RME, the real world is not the formal world of mathematics, but as long as the problems are experientially real in the students' minds ([19]), which means that when teachers explain the approach to students, they need to address that, the scenarios could be in real life situation or could be in people's mind if they are related to their prior knowledge or contents they have learned. When considering realistic, aspects that should be addressed include 1) The use of students' own productions and constructions, 2) Students should be asked to 'produce' more concrete things, 3) Students should be able to reflect on the path they themselves have taken in their learning process, 4) Students may be asked to collect data and draw conclusions 5) Students are asked to design exercises that can be used in a test or to design a test for other students in the classroom ([20]). Therefore, RME builds on students' informal strategies for solving problems and encourages them to draw what they know about a context to model it mathematically. RME will often ask students to discuss familiar scenarios. Contexts can be taken from the real world, from fiction, or from an area of mathematics with which students are already familiar. The important thing is that students can imagine and engage with these scenarios. When students make connections between the real world and mathematical concepts, mathematics becomes relevant to them. The connections between mathematics and real life provide a deeper understanding of the purpose of mathematical concepts and skills. As mathematics becomes relevant, students become more motivated to learn and more interested in the learning process. Real-life situation questions also provide students with an opportunity for a more detailed construction of their understanding as well as a moment to reflect on their learning.

III. RESEARCH QUESTIONS

The purpose of this study is to investigate how to use Realistic Mathematics Education (RME) in mathematic teaching and pedagogical practice, Investigation model and framework are used to elicit students' growth in understanding of mathematics. First, some background information is given about the characteristics of RME related to the Senior mathematic Investigation tasks. Then, the focus is on designing the Investigation tasks based on the information from the theoretical model RME, mathematical contexts and the Australian Curriculum. The power of this framework is that it develops alongside both the teachers and the students: from the survey of students and interviews of teachers, evaluations of the framework are applied after feedback on each task, aiming to provide a valid framework for teachers in designing the Investigation for senior students. This research is expected to highlight the application of REM to Victorian mathematics' new curricula of Investigation, offering students open doors for a guided, innovative, and new approach to exploring and learning senior mathematics confidently. Three research questions are explored during this study, they are:

1. How can Realistic Mathematic Education be applied in the model of Investigation?
2. What pedagogical strategies should be used in designing the Investigation framework?
3. How to revise the Investigation framework based on the results of this study?

IV. METHODS

This study applied qualitative and quantitative research methods. Data collection was conducted through interviews, a survey, and tasks. First, at the start of this study, the first interview was conducted to explore information from teachers 'perceptions and understanding of the mathematics curriculum. The second interview was applied to collect data about how teachers view the Investigation. Analysis of the components of the Investigation model will be discussed and relative information will be recorded, that information will be themed and used as a guide when designing the framework. Second, based on the analysis information, the first Investigation task will be designed and implemented. Third, after completing Investigation One, survey data was gathered on students' views of the Investigation, and the challenges that students faced during the process of doing Investigation One were discussed as open-ended questions. Fourth, comparing the information between teachers' views and students' views will provide directions for future planning. At the same time, the academic results of Investigation One will be analysed and teachers will figure out the weak parts that the Investigation needs to cover or target, also, the academic results provide teachers with further information about students' strengths or needs. After that, Investigation Two task will be designed based on all the feedback and evaluation from Investigation One. To understand more about how teachers adapted to the model and framework of the Investigation, teachers were interviewed about their experience and their developing practice at the end of Investigation Two.

The instrument of research had been validated and revised according to the participants' feedback before being used. Meanwhile, collected data on students' learning outcomes was used to explore the validity of the framework of the Investigations.

This study used RME as a theoretical support to develop practice and use the evaluation protocol and the statistical analysis to provide further details and direction for future planning. In this study, the mathematical practices the students participate in and how these practices are impacted by the progressing activity are considered. Effective pedagogical approaches are applied when designing and revising the components of the Investigation.

V. PARTICIPANTS

61 Year 11 General Mathematic students from a Christian college, which is located in Southeast of Melbourne, participated in the survey and did two Investigations tasks. Three senior secondary mathematics teachers participated in the design, interviews, and evaluation of the modelling of the Investigation. Another 5 mathematics teachers contributed some suggestive advice during the first period of exploring the Australian Mathematics Curriculum about Investigation.

VI. DATA COLLECTION AND DISCUSSION

Question 1: How can Realistic Mathematic Education be applied to the model of Investigation?

Based on the characteristics and curriculum of Realistic Mathematics Education (RME), also consider including skills and strategies that are used in problem-solving and connection with RME, a model is designed (shown in figure 2). The three teachers directly involved in analysing the Investigation task looked through the Australian Curriculum and Victorian Curriculum and decided on the design, knowledge, content, and mathematical skills that would target the components of the Investigation shown in Table I. The draft framework is shown in Figure 3.

The model demonstrated the connections between mathematic knowledge and application in real-life situations, and, provided a guide for teachers when they plan to teach or write the Investigation tasks. After discussion, teachers who participated in the study, summarized some questions for each component, mainly aimed at providing information or guidance for students when they do the Investigation task shown in Figure 3.

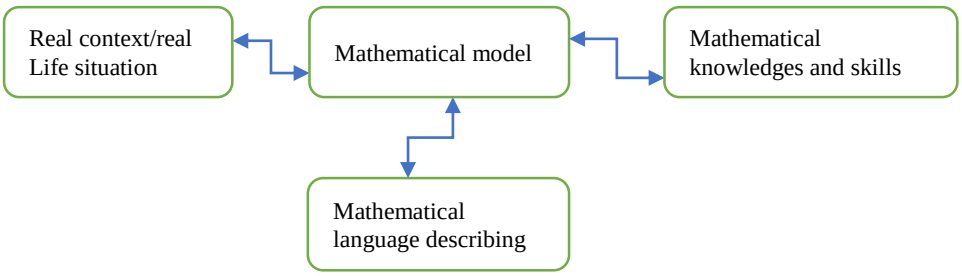

Figure 2. The model of designing senior mathematics

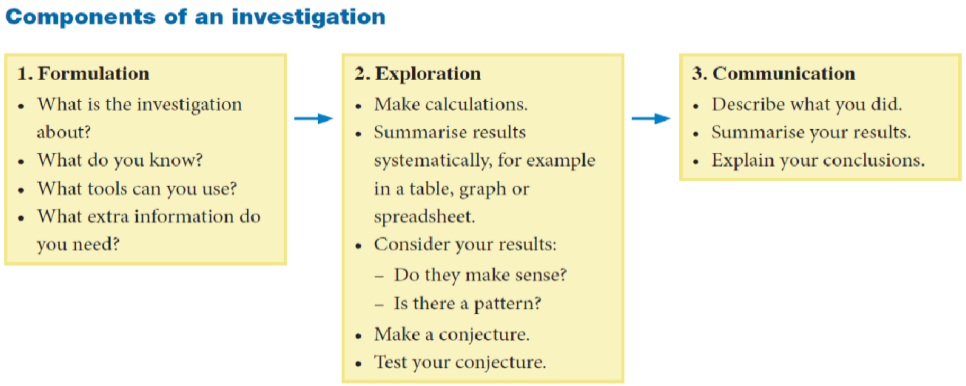

Figure 3. Draft of the Framework of the

Table I. Connections between RME Characteristics and the Investigation

RME characteristics	Investigation
Use the contexts	formulation
Use the model	exploration
Use student's own production or construction/students contribute	exploration
interactivity	communication
Interwinding of various learning strands	communication

Question 2: What pedagogical strategies should be used in designing the Investigation framework?

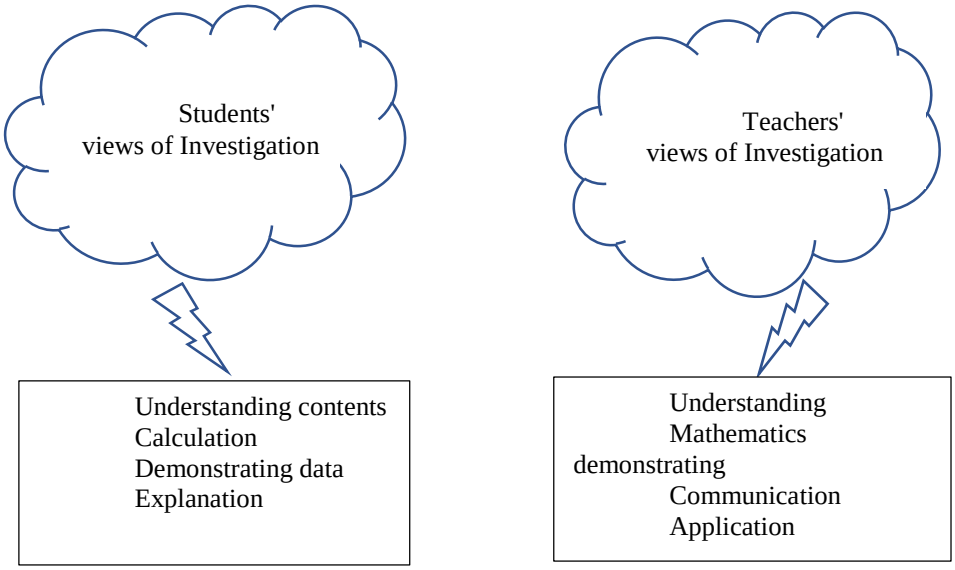

Figure 5. Comparing students and teachers' views of the Investigation

To collect data for research question two, first, the analysis of components of the Investigation and teachers' perceptions of the Investigation were discussed. Participating teachers' views of the Investigation were collected via an interview, while students' views of the Investigation were collected through a survey shown in Figure 5.

To achieve effective teaching of the Investigation, participating teachers discussed applying the pedagogical approaches to addressing the educational skills (Shown in Figure 4). Planning, teaching, learning, assessing, and analysing are included in the Investigation and embedded into the framework. To obtain effective learning, teachers are required in mathematics class to guide students in the arguments, and carefully address the questions that demand thinking ([21]). Skills such as critical thinking, problem-solving, analysing and reasoning are explored during mathematics class. Each component was analysed and explored when writing the Investigation task. Investigation One was about linear modelling.

Figure 4. Pedagogical Approaches of the Investigation

Formulation - "When will I ever use this?" It's a question mathematics teachers hear all the time from their high school students. Teaching mathematics skills is more important than ever ([22]), however, it is often difficult for students to understand the practical applications of this fundamental learning. To provide the Investigation situation 'real' to the students, allowing them to immediately become engaged in the situation, in the formulation section, scenarios related to daily life experiences can provide students with connections to their prior knowledge and experiences, which will help students better understand mathematical concepts and knowledge. Teachers should lead by scenarios and provide students with opportunities to explore different approaches to learning ([22]). For example, in Investigation One, a scenario in the formulation section is as follows:

There is a snagging ceiling in Berry's kitchen. She needs to find a plumber to repair it. She rang company A and was told the charge is $120 on the spot, then charges $80 hourly. Company B charges $160 on the spot and then charges $60 hourly. Berry

wants to compare which company is a good bargain but cannot work it out by herself. Based on the information, you need to help Berry work out the problem. In the project writing, you must demonstrate your understanding and analyse the questions with reasonable maths knowledge.

During the planning section, first, teachers need to consider how to make the formulation part realistic and familiar to the students. The Pedagogy involves several strategies for promoting classroom cultures in which students expect, and are expected to, solve problems for themselves. Strategies in the modelling questions such as 'Say what you see' and asking, 'What else do you know?' as ways of helping students notice what is going on in a particular context or question and think creatively about it. Second, contexts can be taken from the real world, from fiction, or from an area of mathematics with which students are already familiar. Teachers' views of realistic situations are listed in Table II. In addition, the scenario should generate mathematical representations, e.g., equations, tables, graphs etc., which aims to provide students with enough information to generate data and apply those data to the application. Last, the important thing is that students can imagine and engage with these scenarios. Teaching and learning are combined to achieve a satisfactory result only when the contents and methods are in line with students' level and stimulate students' interest.

Table II. Teachers' view of a realistic situation and connection with the Investigation components

Identify real -life event (Formulation)	Mathematics problem-solving strategies (Exploration)	Evaluation (Communication)
Think of a real-life event	Understand the problem. Information you are given. The meaning of the number in the problem.	Using complete sentences, describe your real-life event and the strategy that you used to solve the problem.
Think about the mathematics needed to solve the problem	Devise a plan to solve the problem. Knowledge or skills you can use to solve the problem.	
Ask yourself, "Do I understand what I am trying to find out? Does the event model how I will use mathematics?"	Use charts, graphs, pictures and diagrams when necessary. Implement a plan. Try your strategy, solve it	Explain what you have done. What mathematical knowledge do you use when solving the questions? Demonstrating data is reasonable.
	Reflect on the problem and check if your answer is reasonable.	Tell what worked and what didn't work, if applicable.

Exploration - In the exploration section, first, students are expected to demonstrate the ability to convert word information into mathematical results. Using data, tables, and graphs to show the results of their exploration. Then students are required to write another scenario to provide another similar situation and solve it. The pedagogical theorem involved in exploration are Vygotsky's Sociocultural theory and Piaget's theory of cognitive development. Students are not perceived as "containers" that must be filled with knowledge and skills, they are independent agents of knowledge acquisition ([23]). Beyond the zone of proximal development (ZPD), beyond the comfort zone of their learning, students are taken as powerful learners, who are confident to face the challenge. Students are provided with challenging tasks to promote their learning abilities, capabilities, and cognitive growth. In designing their scenario, teachers further facilitate that students can visualize a mathematical scenario, leading to a stable model that is usable in similar contexts which involves interpreting mathematical problems into pictures or graphics. These connections build students' confidence in thinking about problems which would be the best way of gaining an understanding of the content and skills in solving realistic situations with mathematics knowledge. Furthermore, with appropriate assistance, students will develop a willingness to try ideas out, share these ideas, and listen to others. This flexibility and independence in problem-solving will benefit them greatly in all areas of mathematics and will be important characteristics in promoting learning.

Communication - Communication can help teachers ensure that misconceptions do not become ingrained in students' minds. Students are encouraged to figure things out for themselves, which builds their confidence and capability that they can learn mathematics, they are generous with their own learnings and discoveries. The National Council of Teachers of Mathematics (NCTM) has advocated for its complimenting nature since 1989. NCTM identified mathematical communication as a targeted goal for students ([24]). "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." ([25]), they must be able to organize and communicate their mathematical thoughts, constructing knowledge by interpreting their understanding and making connections to prior knowledge and real-world experiences ([26]). In addition to the importance of communication for mathematic learning, communication as a component is added to the framework due to the interview data that, in mathematics class, teachers do not expect students to elaborate on the meaning of the calculations, students

do not have many chances to practice the skills of communicating mathematically. Also, students were resistant to explaining their understanding of their work in full sentences, and if there were questions that needed to be explained, their descriptions were very brief. A pedagogical approach can be creating a classroom environment where students are allowed to explain their ideas and share their findings, allowing students to reflect on their interpretations, recognize conflicts and contradictions, and consider the mathematical qualities of specific strategies and models. Teaching strategies therefore include activities that engage students in analysis, practice, listening, reflection, argumentation, and discussion ([27]). Therefore, in the Investigation, written communication is included as a vital part for teachers to assess students' learning and understanding. Students are expected to understand the process, and the relationship between teaching and learning, to clarify their decisions and conclusions.

Question 3: How to revise the Investigation framework based on the results of this study?

1) *The survey of students' view of Investigation One task:* REM helps students to make sense of their intuitive strategies, like reading and comprehension, and relative mathematical contents, analyse the scenarios and calculation skills. Through this process, students gain ownership of their mathematical knowledge and a deep understanding of where mathematics comes from. After Investigation One, a survey was applied to collect data about students' experiences of the Investigation, including students' views of the Investigation, difficulty level and confidence about doing the Investigation.

61 students participated in the survey and 55 students provided data about how they felt about the mathematical Investigation. When they were asked what the Investigation was about? From the data (shown in Figure 6), the top two were 'understanding mathematical contents' (77%) and 'calculation skills' (77%), and 75% of students chose 'demonstrating data'. The first three preferable choices matched teachers' views of the Investigation, the data also showed that students understood the aim of the Investigation and could list most of the key criteria that teachers expected them to master. However, 'Connecting school mathematics knowledge to practical situations' (64%) and 'Demonstrating my understanding of the realistic situation' (59%) which were the two important skills that RME addressed, were in a low ranking. The data demonstrated that students are not very actively involved in the process of learning ([18]) and they only take learning as an achievement not thinking and expressing themselves

When students were asked 'How do you feel about the Mathematical Investigation?' 12.7% of students felt they were highly confident, 60% of students felt they were mostly confident about taking the Investigation, there are 23.6% of students felt partially confident about doing the Investigation, and 3.6% of students feel not confident at all. The data showed that most students are confident to do the Investigation after the introduction of what this task is about, and the structures that students can follow. Comparing the percentage of student's confidence in the survey with the academic results in Investigations One and Two, data shows that students' level of mastering mathematical knowledge and skills or their confidence in learning, and challenging themselves in mathematics influenced their responses to the survey.

Figure 6. What the Investigation is about?

Table III. Students' Views of Challenges

Highly confident	Mostly confident	Partially confident	Not confident
12.7%	60%	23.6%	3.6%

2) Evaluation of Investigation One: After students completed Investigation One, teachers were interviewed to get feedback on the task. Investigation One is about linear relations and modelling. Interview data were analysed thematically, focusing on teachers' accounts of the connections between the Investigation design and students' achievement. Improvements or concerns were also discussed during interviews.

Evaluation from teachers: 1) Investigation stretches and strengthens the mathematical knowledge and skills that are useful or relevant to students' future lives or careers; cultivating students' learning habits in analysis, and critical thinking and being able to choose, pursue and evaluate learning by themselves. 2) Based on Investigation One, students simply have difficulties in the communication section, like what they have learned and how to explain their learning in detail. Students need different kinds of Investigation topics involving problem-solving and synthesis skills. Overall, all the efforts they make in doing the Investigation are fruitful. 3) The Investigation is designed to support fundamental skills while developing a deep understanding of key concepts of the content. With the teacher's guidance, students can develop and articulate their own models of problem scenarios and bring these together in ways that give long-lasting meaning to key concepts of linear modelling. Students need to improve their sharing, explaining, and discussing in problem-solving.

Improvement: 1) When writing the questions of the Investigation, how to demonstrate the data need to be specified for students. 2) Data collection is suggested to be added as a component. Because this will help students understand how to collect primary data, help students understand what the data means. The more steps students get involved in, the better they will understand the process of the Investigation. 3) Increasing the size of samples to get more details of the data. 4) Increase more samples from students' real lives to increase the reliability, also this can engage students in their learning.

3) Compare the two Investigation tasks and analysis of students' academic results:

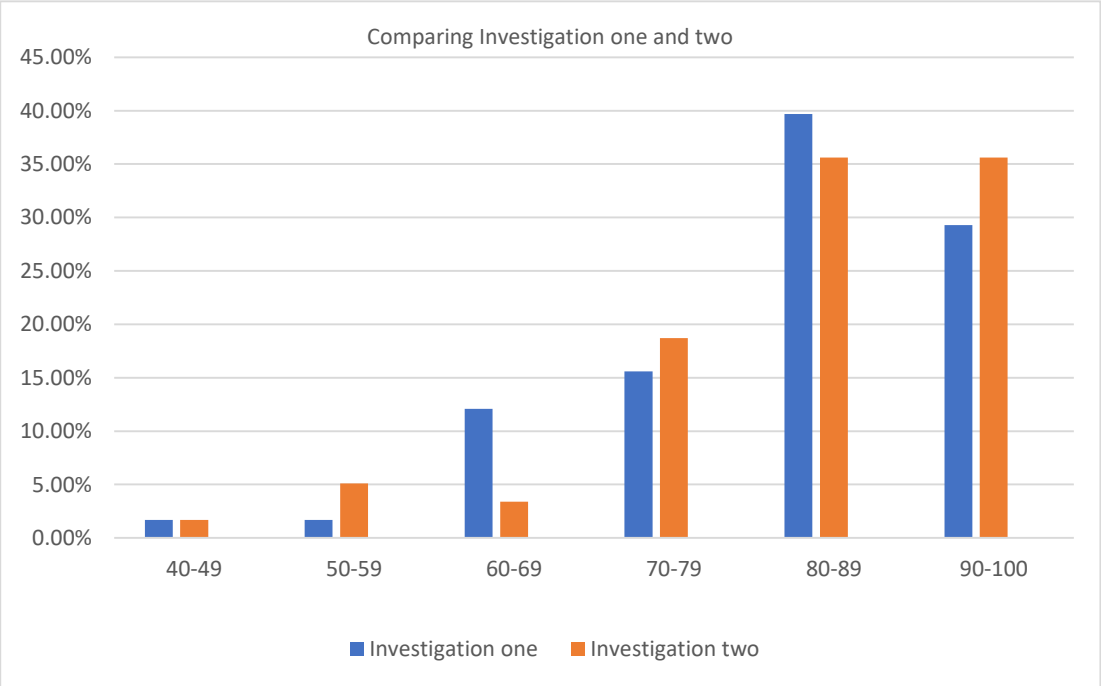

Figure 7. Comparing students 'academic results of the Investigation One and Two

Investigation Two is about relationships between two numerical variables. One finding (shown in figure 7) is that the number of students who achieved over 90% of the Investigation has increased from 29.3% to 35.6%, which means students who are at the high level make a jump to achieve satisfactory results, students who have strong mathematical knowledge and skills, improved obviously than the others. Another finding is that the number of standard-level students who achieved over 70% has increased from 15.6% to 18.7%, which demonstrates that if teachers "trying to put the learning in a context that the kids care about and that makes sense to them" ([14], p547), providing an opportunity for students to build on the knowledge, students will show their gain. There are 1.7% of students who are 'weak 'in mathematics learning, they made some mistakes during Investigation Two, such as recognising quantities that influence the given situation, identifying the key variable; and failing to construct relations between variables. Teachers, as markers, need to be aware that, the aim of the Investigation is, after applying RME in the tasks, to address the importance of 'real world' questions for 'weaker' students to identify the connection between mathematics learning and their daily life ([14]), to get them to the point where they can function in the real-life situation, thus, it is necessary to push students from their 'comfort zone', to inspire them and facilitate them in the correct track of their mathematics learning.

When analysing the two Investigation results, from the students' answers can find out that the majority of students demonstrated understanding of the questions and could provide the correct answers, however, analysis in the communication section showed that students demonstrated their understanding mainly due to the memorize how the formulas are written in a textbook along with the examples, they can follow the instructions do the work properly. When they were asked to explain what they did, learned and summarised their results and found, that most students did not show good communication skills. This finding highlighted that in future mathematics lessons, teachers should address or add activities that can inspire students to get involved in proposing opinions, and explanations, generating discussion and comparing the similarities and differences between his/her answer to others. Another finding out is that there are several types of errors in solving problems related to algebraic material, such as misunderstood responses and explanatory variables, a wrong modelling of the relationships between the two variables, and a wrong regression linear equation, which indicates that student's mathematical demonstrating skills have not been well developed ([28]). More routine practice questions will develop students' mathematical modelling abilities ([1]). Furthermore, teachers need to facilitate students who are weak in transferring worldly descriptions to mathematics expressions.

4) Evaluation of Investigation Two: 1) Delivering a reality-based curriculum and pedagogy will motivate students to get involved in the learning process. RME had positive impacts on student engagement, understanding and approaches to problem-solving. Teachers may play the role of 'master' but should be supportive. Rather than teaching as experts, teachers should play the role of facilitators, conveying knowledge with real-life examples and frequently and clearly promoting the authority of further research in mathematics, consequently, many students can get rid of the difficulties in mathematics learning. 2) Teachers should be aware of classroom learning and the need for building a connection with realistic problems is significant, teachers can use this information to influence and guide students in their learning. 3) Teachers need to decide on which chapter/content is suitable for adapting the Investigation. Providing realistic examples, explaining the expectations of the Investigation, and skills of application. Teachers need to make sure that students understand which mathematics skills are expected to take, what they have learned from the classroom to the situation that they need to deal with. 4) For the 'weak' students, allow students to learn correlatively. Design an activity for students to practice the skills related to individual components of Investigation, for example, given students questions, they can discuss as a small group, and they will come out with what information they need to gather to answer the questions, rather than do the whole Investigation, which would be supportive for students who lack confidence in learning mathematics.

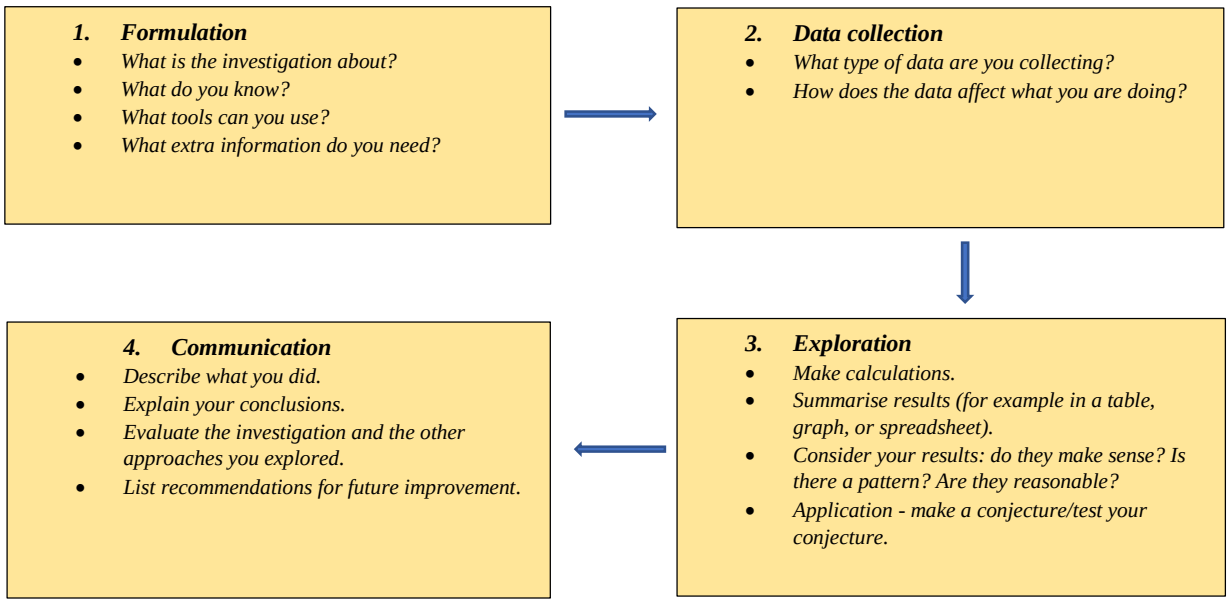

Figure 8. Components of the Investigation(revised)

5) Revise the components of the Investigation Framework: based on the data and information from interviews, surveys, and students' academic results of the two Investigations, how to revise the framework was discussed. Data from Table IV showed that there is no significant difference between the two investigations' academic results, which means that the components work well with the topic of the Investigation, students also demonstrate a good understanding of the Investigation and confidence in facing the challenge of the task. Learning mathematics in the classroom or other learning settings has been various. Teachers' designing and planning as a field of inquiry and practice better develop as a half-and-half between the spaces of teaching and practising ([29]). Based on RME characters, effective teaching and learning, and pedagogical approaches, such as constructive theory and social-historical theory, which address the educational skills for effective learning were explored in the design. The components of the Investigation also consider students' learning outcomes to reflect the effectiveness of the pedagogical approaches applied in this design. Questions in the exploration part were refined, and the aims or methods that students need to consider when answering questions were specified.

Considering the standardization of the framework of the Investigation, data collection as a component was added, the revised one is given in Figure 8.

Table IV. Academic results of the Investigation One and Two

	Academic results			
	Class A	Class B	Class C	Average
Investigation One	83.20	82.33	80.19	81.8
Investigation Two	83.18	81.27	82.37	82.3

VII. CONCLUSION

Summarize the findings: REM emphasizes that mathematics is a human activity, this character was remembered and used as a guide when designing the components of the Investigation. This study combined pedagogical theorem with the Australian Curriculum and Victorian Curriculum, as well as embedded 21st-century skills within the design of the investigations, skills such as critical thinking, creativity, research and inquiry, self-direction, information use, communication, and reflection ([30]). Mathematics learning should occur in a format that allows students to reinvent ideas and mathematical concepts through exploration under teacher guidance. The effects of this design on students' attitudes, problem-solving abilities, and learning interests are demonstrated. Effective learning will occur only if teachers understand the skills and consider a particular teaching method. The Investigation is an effective type of task for teachers to evaluate their planning, and for students to explore and assess their learning. When students work in meaningful contexts like these, they do more than just learn specific mathematical skills. By learning to examine shared mathematical relationships between context and real-life situations, students connect their experiences to active ideas about the nature of certain mathematical concepts.

The study found that the framework of the Investigation worked effectively, and students demonstrated confidence in doing the task, they also showed their understanding of the aim of the task and achieved great academic results for the two Investigations. In this study, students applied mathematical representations including graphs, tables, symbols, mathematical notes, written text representations, words, and language, which matches the founding of another research ([31]). This study unveiled students will learn mathematics confidently when they build the connection between mathematics learning and their life situation; Investigation is an effective assessment method to help teachers examine the effect of RME on improving mathematical representation ability ([32]). The study also showed that students need more opportunities to participate in the learning process, such as proposing opinions, expressing their own ideas or methods, engaging in discussion, and presenting data. This study highlighted the need to improve students' mathematics communication skills for effective learning to occur. The model and framework designed in this study will guide teachers in designing Investigation tasks that connect mathematics learning at school and real life, thereby motivating students to learn mathematics actively, positively, and confidently.

Limitations: There is progress compared to the first Investigation, this is a hypothesis estimate of the improvement of the achievement, however, there is no significant difference between those two Investigations. Because of the time limit, only two Investigations were explored, considering this, more participants got involved in the framework can achieve better outcomes of reliability. The framework needs to be implemented more times in future to test its validity. Also, a longitudinal practice to verify the framework's reliability is needed in future.

Future research areas: Future research areas will be worth exploring including research about the implementation of the Investigation framework, not only for senior students but also junior students; research about how to improve students' mathematical communication skills; research about the relationships between students' academic progress and the application of RME, assessing the impact of the RME approach on mathematics attainment; RME pedagogy and problem-solving skills; RME pedagogy and mathematics curriculum, also, research about the influence of RME programming training between service teachers' confidence in mathematics teaching will be a good area to be investigated.

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