

## The Use of Synergetic Methods and the Catastrophe Theory



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**ABSTRACT:** The state of the global ecosystem is currently approaching a critical point, which affects the socio-economic condition of the society. At the present time, the subject of study of economics and financial sciences should be the study of a world characterized by non-stationary changes; socio-economic and ecological crises, which in themselves are related to the multidimensionality and non-linearity of socio-economic systems. The article builds various models using the theory of catastrophe in the process of global socio-economic development, The Ways of solving them are set, the raised models are solved by the computer program Maple, solutions are given and three-dimensional graphs are constructed. For the purpose of analysis, we have selected a synergetic methodology based on the theory of self-organization of complex systems. Various methods and tools were discussed for modeling the unsustainable development of socio-economic systems of which conceptual-methodological basis is catastrophe theory. The algorithm is defined and a complex of models is built in the article, which allows us to determine the type of macroeconomic indicators, the nature of the dynamics, and to determine the possibilities for the development of crises. In the 21st century, it has become clear that the state of the global ecosystem is approaching to a critical bound and it is reflected on the socio-economic development of the society, in addition, the Covid 19 pandemic made the situation extremely tense. Gradually, we are already approaching to catastrophe with big steps. Given this situation, these types of studies are of particular importance.

**KEYWORDS:** bifurcation, macroeconomic indicators, global crisis, synergetic, socio-economic development, the catastrophe theory, financial system, cycles.

### THE MAIN PART

Economic and financial systems are discussed as a structural condition of society and precisely, it is typical for them the volatile, erratic, crisis, and catastrophic development full of dynamism. In these conditions, security is especially important and relevant for individual regions, as well as for specific states and for the global economic and financial system in general. In these conditions, security and sustainability issues are particularly important and relevant for individual regions, as well as for specific states and for the global economic and financial system in general, which should be discussed as a certain skill, regulatory, in terms of the movement of certain finances and resources all these indicators represent the parameters for determining and studying the functioning of the system. In the scientific community the idea is already gaining a foothold to the direction to create new concepts, methodological and methodical bases, the new methods of analysis, dynamic methods relevant to the current situation, which will have flexible means of impact, and correspond to the stage of development of the self-organizing system. Dynamic models that have already worked well in biology, physics, catastrophe theory, are the ways to solve the problems of forecasting finances and the financial system as a whole. It is a science on the verge of topology and mathematical analysis that analyzes the behavior of nonlinear dynamic systems under conditions of change in their parameters. Papers have already appeared where the forecasting of life development on Earth is discussed in the perspective of modern physical science. Work has been carried out to establish a mathematical apparatus for the theory of cyclical development of catastrophe. Therefore, it was possible to describe the basis of the physical-mathematical methods of the evolutionary development of society, where the elements of the theory of chaos were developed. Based on such studies, the term synergetic has been proposed to identify dissipative structures. Today these methods are not limited solving the tasks of the physical sciences, but the field of synergetic approaches has greatly expanded and it is successfully used in other sciences, particularly in economics and finances. In today's conditions, such approaches are widely used in the strategic planning of global financial problems that humanity faces.

In this article, we use synergistic methods based on the theories of complex systems self-organization, self-reorganization, and self-management. Based on the synergetic theory in finances and in economics issues, it is necessary to examine the dynamics of economic and financial issues. In this case, the focus is on systems development, growth, systems disorganization, self-organization and connectivity issues. In the research process it is necessary to focus on the environment as a source of usable

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parameters and volatile situations. As a result, we must investigate the internal and external connections of the research system to these parameters and at the same time we must take into account the fact that chaos in the system causes not only destruction, but also contributes to the development of the system. In the previous century, scientists have already shown in their work that socio-economic development cannot be monotonously growing. Sooner or later there comes a moment when the development of the economy and the financial system enters a crisis phase. The causes of the crises are still unknown, and discussions are ongoing. Hopefully, when we identify the causes of crises, it will allow us to predict future crises and avoid them as much as possible.

Today, the world and every country is facing the most difficult economic crisis, which is caused by the imbalance of the economy, the inequality of socio-economic development, and the sharp decline caused by the Covid-19 pandemic. Mankind does not remember a catastrophic situation of a similar magnitude. As a result, the danger of disintegration is expected. A crisis is forming, both at the regional and national levels. Economic crisis causes serious changes, the movement of various resources, including financial resources, which has a negative impact on the socio-economic situation.

Lack of ability to predict global, national, regional crises and catastrophes urgently requires refinement of economic and management methods. The most promising tool in this direction is the theory of catastrophe and the direction of synergetic research. Mathematical and catastrophic modeling theory of cyclical development is already being implemented and needs to be further expanded. It is necessary to create a physical-mathematical theory of society development, in which we will also use theories of catastrophe and chaos. The socio-economic system itself is a multi-level, multi-stage system. When defining or applying initial parameters, any uncertainty, or a random probabilistic situation, at different levels cause unexpected large changes at the lower level. Considering the above factors, we can say that the system contains catastrophe or signs of catastrophe. The existence of potential, critical points in the description of the system is also confirmed by the fact of catastrophe. From the signs that contain catastrophe, we can single out the following signs:

- **Modality** - This is a feature of the system, characterized by the fact that for some the meaning of the control parameters several equilibrium states are expected in a certain area of attribution of these parameters.
- **Unattainability** - This is a state of equilibrium that is impossible to achieve or is unattainable to move from one state of equilibrium to another. (There is an area of unstable equilibrium, when we come out from a sustainable state).
- **Catastrophic jumps** - This is the transition from one sustainable state of the system to another. (Even minor changes of the managing parameters can cause large changes to the system settings values, due to the system jumping from one local minimum to another).
- **Mismatch** - This is a small change in the parameter space that leads us to a state significantly different from the desired final state of the system.
- **Hysteresis** - is the transition from one state of the system to another and return the control parameters in other meaning of conditions.

If one of the signs of a catastrophe will be detected during the system analysis, then by changing the control parameters, it is possible to detect other signs of an impending catastrophe as well. When modeling ongoing processes in socio-economic systems and subsystems, it is necessary to consider the following suggestions:

- The state changes in time (dynamic system requires dynamic model):
- **The principle of maximum delay:** The system tries to maintain its condition as long as possible. Additional modeling is necessary for the researcher, or the situation should be assessed from the beginning and this method which will be dragged in time and it will be carried out to the point of bifurcation;
- **Current state of the system** depends on how the system came to this state. It is necessary to investigate the factors of the previous period (before the catastrophe) that brought the system to this state;
- **When making a change:** When we change the control parameters of the system in a strongly opposite direction, we need to know that the system can no longer return to its original state, because the system is nonlinear and multidimensional and the trajectories of the system are irreversible.

Catastrophes are most often discussed in economics, the dynamics of which are generally given by the following equation:

$$\ddot{x} = \nabla V(x, \alpha)$$

Where  $V(x, \alpha)$  is a potential function;  $x$  – the phase coordinates of the system;  $\alpha$  is the vector of parameters. The study is performed when changing the control parameters to bring the potential function out of equilibrium. The study is performed when changing the control parameters to bring the potential function out of equilibrium. In this case the surface of the catastrophe is defined as the set of equilibrium points (equilibrium surface) and is given by the following ratio:

$$M = \left\{ (x, \alpha) \in R^n \otimes R^k : \left( \frac{\partial V}{\partial x} \right) = 0 \right\}$$

Where  $R^n, R^k$  – are  $n$  and  $k$  which are dimensional Euclidean spaces.

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And the critical points  $\det = \left( \frac{d^2V}{\partial x_i \partial x_j} \right) = 0$  are singular dots i.e.

$$S = \left\{ (x, \alpha) \in R^n \otimes R^k : \det = \left( \frac{d^2V}{\partial x_i \partial x_j} \right) = 0 \right\}$$

The projection of the singularity set on the parameter space is a bifurcation set:

$$B = \{ \alpha \in R^k : V_{xx} = 0 \}, \text{ where } V_{xx} = \left( \frac{d^2V}{\partial x_i \partial x_j} \right)$$

If the potential function depends on several control parameters, then the matrix of sustainability  $V_{xx}$  and its eigenvalues depend on these parameters. In this case it can be said that one or more of the property values of the stability matrix on some of the values of the control parameters may be equal to zero. In this case it becomes impossible to represent the potential function as a square shape. However, it is possible to make some divisions that allow us to find coordinates that correspond to zero eigenvalues:

$$V(x, c) = \text{Cat}(l, k) + \sum_{j=l+1}^n \lambda_j(c) y_j^2$$

Where  $\text{Cat}(l, k)$  is a function of the catastrophe.  $l$  - is the eigenvalues of the sustainability matrix under certain additional conditions.

$$V = CG(l) + \sum_{j=l+1}^n \lambda_j y_j^2$$

Where  $\text{Cat}(l, k) = CG(L) + \text{Pert}(l, k)$ ;  $CG(l, k)$  is catastrophic increase.  $\text{Pert}(l, k)$  – Concern. Table 1 shows the classification of potential functions (catastrophes) and their types. The presented algorithms for catastrophic models of macroeconomic indicators include:

- The identification of the system of macroeconomic indicators temporarily scattered on the horizons and its interaction;
- Assessment and analysis of catastrophe types of identification systems;
- Especially the construction and analysis of surfaces of probable catastrophes.

According to the algorithm of the analyzed indicators, we built certain model complexes for the types of catastrophes indicated in the table. In some of them (folds, tuft, swallowtail, butterfly, wigwam) they are related to the unstable relationship of one  $x$  variable with other variables and at times of ombilian (elliptical, hyperbolic, and parabolic) catastrophes are associated with other variables in case of instability of the investment growth  $x_1$  and industrial production growth  $x_2$  variables. In the study of economic processes, it is of particular interesting the study of investment and industrial activities for crises that have been already experienced or is expected in the economies of different countries. The model below approximates the  $x$ -rate of investment growth, with the gross domestic product (GDP) variable  $y$ .

**Table 1**

Disaster type	Number of parameters	Canonical form	Equilibrium surface	Abundance of singularity	Bifurcation abundance
Fold type	$l=1, k=1$	$V(x, u) = x^2 + ux$	$M: 3x^2 + u$	$6x=0, x=0$	$U=0$
Bundle	$l=1, k=1$	$V(x, u, v) = x^2 - ux^2$	$M: 3x^2 + u=0$	$M: 12x^2 - 2u$	$B: 8u^3 - 27v^2=0$
Swallow tail	$l=1, k=3$	$V = x^5 + ux^3 + vx^2 + wx$	$M: 5x^4 + 3ux^2 + 2vx + w = 0$	$S: 20x^3 + 6ux + 2v = 0$	$B: 3x: 5x^4 + 3ux^2 + 2vx + w = 0, 20x^3 + 6ux + 2v = 0$
Butterfly	$l=1, k=4$	$V = x^6 + ux^4 + ux^3 + ux^2 + ux$	$M: 6x^5 + 3ux^2 + 2vx^2 + w = 0$	$S: 30x^4 + 12x^2 + 6ux + 2v = 0$	$B: 3x: 6x^5 + 3ux^2 + 2vx^2 + w = 0, 30x^4 + 12x^2 + 6ux + 2v = 0$

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Vigvam	$l=1$ , $k=5$	$V = x^6 + a_1x^5 + a_2x^4 + a_3x^3 + a_4x^2 + a_5x$	$M: 7x^6 + 5a_1x^4 + 4a_2x^3 + 3a_3x^2 + 2a_4x + a_5 = 0$	$S: 42x^5 + 20a_1x^3 + 12a_2x^2 + 6a_3x + 2a_4 = 0$	$B: 3x: 7x^6 + 5a_1x^4 + 4a_2x^3 + 3a_3x^2 + 2a_4x + a_5 = 0, 42x^5 + 20a_1x^3 + 12a_2x^2 + 6a_3x + 2a_4 = 0$
Hyperbolic observation	$l=2$ , $k=3$	$V(x, y, u, v, w) = x^3 + y^3 + wxy - ux - vy$	$M: \begin{cases} 3x^2 + wx - u = 0 \\ 3y^2 + wx - v = 0 \end{cases}$	$S: \det \begin{vmatrix} 6x & w \\ w & 6y \end{vmatrix} = 36xy - w^2$	$B: 3x: u = 3x^2 + wx, v = 3y^2 + wx, 36xy - w^2$
Elliptical observation	$l=2$ , $k=3$	$V(x, y, u, v, w) = \frac{x^3}{3} + xy^3 + w(x^2 + y^2) - ux - vy$	$M: \begin{cases} x^2 + y^2 - wx - u = 0 \\ -2xy^2 + 2wy - v = 0 \end{cases}$	$S: \det \begin{vmatrix} 2x + 2w & -2y \\ -2y & -2x + 2w \end{vmatrix} = \Rightarrow w^2 = x^2 + y^2$	$B: 3(x, y): u = x^2 - y^2 + 2wx, v = 2xy + wx, w^2 = x^2 + y^2$
Parabolic observation	$l=2$ , $k=4$	$V = x^2y^2 + y^4 + ax^2 + ty^2 - ux - vy$	$M: \begin{cases} 2xy + 2ax - u = 0 \\ x^2 + 4y^3 + 2ty - v = 0 \end{cases}$	$S: \det \begin{vmatrix} 2y + 2w & -2x \\ 2x & 2y^2 + 2t \end{vmatrix} = 0 \Rightarrow (y + w)(y^2 + t) = x^2$	$B: 3(x, y): 2xy + 2ax - u = 0, x^2 + 4y^3 + 2ty - v = 0, (y + w)(y^2 + t) = x^2$
<b>u, v</b> - Options; <b>w</b> – Sustainability ; t - Time					

$$y = x^7 - 3,746x^5 - 0,318x^4 + 3,478x^3 - 0,355x^2 - 1,552x$$

This equation can be solved in program Maple as follows:

restart

$$y := x^7 - 3.746 * x^5 - 0.318 * x^4 + 3.478 * x^3 - 0.355 * x^2 - 1.552 * x = 0$$

S:=solve(y,x)

As a result we got seven roots. The true roots are marked in green complexion with pink.

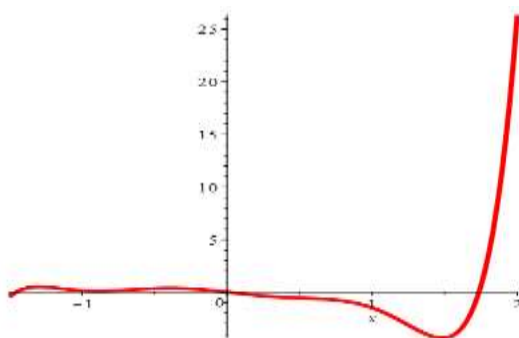
$$x_1 = 0.; x_2 = 1.736016018; x_3 = .7560278284+.4103479221*I;$$

$$x_4 = .8972105348+.1600164122*I; x_5 = -1.453650605; x_6 = -.8972105348-.1600164122*I;$$

$$x_7 = .7560278284-.4103479221*I$$

The graph of this function looks like this:

plot( $x^7 - 3.746 * x^5 - 0.318 * x^4 + 3.478 * x^3 - 0.355 * x^2 - 1.552 * x$ , x = -1.5 .. 2, thickness = 4, color = red)



restart

```
pp := solve({7*x^6 - 18.736*x^4 - 1.272*y + 10.461*x^2 - 0.7*x - 1.552 = 0, 0.42*x^5 - 74.92*y
- 3.81*x^2 + 20.92*x - 0.71 = 0}, {x, y})
{x = 1.430938944, y = 0.3195898786}, {x = 0.5733892997 + 0.1721876638 I, y
= 0.1354677852 + 0.03846688496 I}, {x = -0.3778432859, y = -0.1222858087}, {x =
-0.8282675205, y = -0.2778275493}, {x = -1.370588050, y = -0.5148315224}, {x
= 0.5733892997 - 0.1721876638 I, y = 0.1354677852 - 0.03846688496 I}
```

Fig.1

The model of "wigwam" catastrophe looks like this:

$$\begin{cases} 7x^6 - 18.736x^4 - 1.272y + 10.461x^2 - 0.7x - 1.552 = 0 \\ 0.42x^5 - 74.92y - 3.81x^2 + 20.92x - 0.71 = 0 \end{cases}$$

Solving this model in the program Maple. Fig. 1

The surface of a given catastrophe looks like this:

Fig.2

```
plot3d ({.42*x^5-74.92*y-3.81*x^2+20.92*x-.71, 7*x^6-18.736*x^4-1.272*y+10.461*x^2-.7*x-1.552}, x = -10 .. 10, y = -1 .. 1)
```

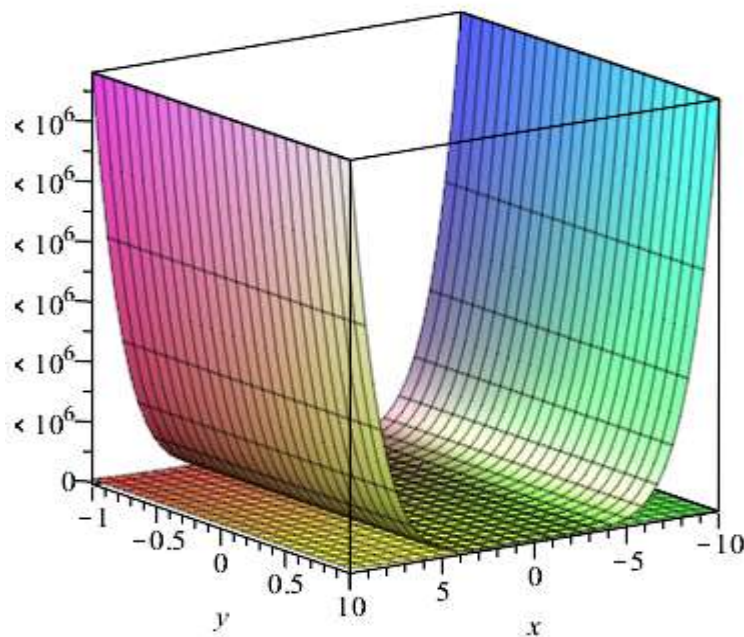


Fig.2. Similarly, a computer analysis of any of the catastrophe listed in Table 1 can be performed.

**CONCLUSION**

In the current existing condition of mankind, the research presented in this article allows us to make a timely prediction of catastrophe and a comprehensive analysis of the situation. The models in the article are easily implemented on a computer, allowing us to prevent an impending economic catastrophe. It should also be noted that computer-implemented math programs Maple, Matlab, and others make it much easier for us to make the real-time predictions and avoid catastrophes.

**LITERATURE**

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